Optimum Design of Multiple Configuration Structures for Frequency Constraints

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Introduction

DURING the service of an aerospace structure, its configuration may vary for different operating conditions. When we restrict the word "configuration" to structural geometry, the structures with multiple configurations that come to mind are, for example, a tilt-rotor aircraft in airplane or helicopter mode and a stowed vs deployed solar array system for a spacecraft. If we generalize the word configuration to encompass boundary conditions and mass distribution, then all aerospace vehicles can be considered to be structures with multiple configurations.

In application, design specifications may impose specific natural frequency constraints for different configurations. For a structure with a single configuration, the minimum weight design with frequency constraints can be found readily.[1-4] However, the resulting optimum design may not satisfy frequency constraints for the structure in other configurations. This often leads to design iterations among various configurations. After many design iterations, the final design may be feasible but not optimum. The purpose of this paper is to develop an approach for minimum weight design of structures with several configurations under natural frequency constraints. In the optimum design problem, frequency constraints for all configurations are considered simultaneously.

In each design cycle, the constrained minimum weight design problem is solved iteratively by a combined analysis/optimization procedure. To increase the computational efficiency, the iterative analysis is approximated by using reduced-order modal space models.[4,5] Sequential linear programming (SLP)[2,4,5] for optimization procedure is considered. In SLP, the optimum design problem is linearized at each design iteration. Together with a strategy to compute move limits, the resulting linear optimization problem is solved as a linear programming problem. The combination of finite element analysis together with optimization method (SLP) constitutes an effective and reliable approach for solving practical optimum design problems.

Problem Statement

For multiple configuration structures, different frequency constraints may be imposed for each configuration. The optimum design problem can be stated mathematically as find design variables X_i, $i = 1$ to N to minimize the structural weight

$$W = \sum_{i=1}^{N} C_i X_i$$

subject to

1) Natural frequency (eigenvalue) constraints:

$$\lambda_{j,L}^{(m)} \leq \lambda_j^{(m)} \leq \lambda_{j,U}^{(m)} \tag{1}$$

2) Side constraints:

$$X_{i,\min} \leq X_i \leq X_{i,\max} \tag{2}$$

where C_i is objective function coefficient. In constraint 1, $\lambda_{j,L}^{(m)}$ and $\lambda_{j,U}^{(m)}$ are lower and upper bounds, respectively, for the natural frequency of the jth mode in the mth configuration.

Proposed Approaches

There are many approaches in the literature that can be used to solve the optimum design problem. Sequential linear programming with move limits has been implemented here. They are described as follows.

When the natural frequency $\lambda_j^{(m)}$ is linearized by first-order Taylor series expansion, the constraints 1 become linear constraints in X_i, and the previous optimization problem becomes the linear programming problem.

To ensure convergence of the SLP iteration, we also need to impose the following move limits:

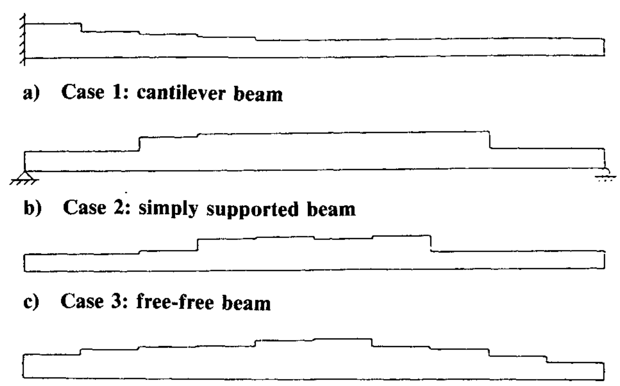
$$X_{i,L} \leq X_i \leq X_{i,U} \tag{3}$$

where $X_{i,L}$ and $X_{i,U}$ are lower and upper bounds of X_i defined by the following adaptive procedure.

The bounds on design variables can be computed as

$$X_{i,L} = X_i - \Delta X_c \tag{4}$$

$$X_{i,U} = X_i + \Delta X_c \tag{5}$$



a) Case 1: cantilever beam

b) Case 2: simply supported beam

c) Case 3: free-free beam

d) Case 4: three different models together

Fig. 1 Final design for four cases.

Table 1 Description of design cases for numerical example

Case	Model	Constraint
1	Cantilever beam	$\lambda_1^{(1)} \geq 9.0$ Hz
2	Simply supported beam	$\lambda_1^{(2)} \geq 26.5$ Hz
3	Free-free beam	$\lambda_3^{(3)} \geq 58.0$ Hz
4	Cantilever beam	$\lambda_1^{(1)} \geq 9.0$ Hz
	simply supported beam	$\lambda_1^{(2)} \geq 26.5$ Hz
	free-free beam	$\lambda_3^{(3)} \geq 58.0$ Hz

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Table 2 Final results of numerical example

Initial			Final			
Case	Weight, lb.	Frequency, Hz	Weight, lb.	λ'(1)[a] ≥ 9.0 Hz	λ'(2)[a] ≥ 26.5 Hz	λ'(3) ≥ 58.0 Hz
1	25.13	λ(1) = 8.55	19.60	9.02	(20.47)	(42.80)
2	25.13	λ(2) = 24.12	28.11	(7.49)	26.52	(72.39)
3	25.13	λ(3) = 53.05	22.17	(7.34)	(22.93)	58.37
4	25.13	λ(1) = 8.55 λ(2) = 24.12 λ(3) = 53.05	28.02	9.02	26.52	67.55

[a]Note: λ(1) and λ'(1) are initial and final fundamental natural frequencies for the beam with boundary conditions of case 1.

Table 3 Final design of numerical example

Member	Case 1, in.^{2}	Case 2, in.^{2}	Case 3, in.^{2}	Case 4, in.^{2}
1	3.56	2.19	2.19	2.85
2	2.73	2.75	2.19	3.32
3	2.69	4.07	2.52	3.69
4	2.32	4.07	3.55	3.81
5	2.19	4.07	3.65	4.31
6	2.19	4.85	3.32	4.44
7	2.19	4.13	3.65	3.87
8	2.19	4.22	2.19	3.69
9	2.19	2.54	2.19	2.82
10	2.19	2.19	2.19	2.19
Weight, lb.	19.60	28.11	22.17	28.02
No. of cycles	2	1	1	2

where maximum move limit

$$\Delta X_c = \mathrm{MAX}(\Delta X_j), \qquad j = j\text{th constraint} \tag{6}$$

By linking all design variables together, ΔX_j can be computed from the condition

$$g_j = g_j(X_{io}) + \sum_{i=1}^{N} \frac{\partial g_j}{\partial X_i} \Delta X_i = 0 \tag{7}$$

where $g_j = j$th constraint and we assume $\Delta X_i = \Delta X_j$, $i = 1$ to N. Let

$$\Delta X_j = \left| \frac{g_j(X_{io})}{\sum_{i=1}^{N} \frac{\partial g_j}{\partial X_i}} \right| \tag{8}$$

The solution of the linear programming (LP) problem provides an updated design that can be analyzed by using assumed-mode reanalysis procedure in Structural Optimization and Reanalysis (SOAR). A new LP problem can then be formulated and solved in each iteration. When the SLP procedures converge, the models are updated and finite element analysis is performed on each to provide data for next design iteration.

Numerical Example

We seek to find the minimum weight design of a beam with constraints on the fundamental natural frequency under different boundary conditions. The beam has a circular cross section and a length of 80 in. The beam material is aluminum with specific weight 0.1 lb/in^3, modulus of elasticity 10×10^6 psi and Poisson's ratio 0.3. Four cases have been solved, and the results compared. These four cases are defined in Table 1. In the first three cases, the beam is optimized with a single configuration. In case 4, the frequency constraints for all three configurations are considered simultaneously. A uniform beam with cross-sectional area equal to 3.1416 in.^{2} is used as the initial design. The beam is divided into 10 elements. In the subsequent designs, the cross-sectional area of each element is treated as an independent design variable, with a lower bound of 2.199 in.^{2}

The natural frequencies of the final design are shown in Table 2. The corresponding optimum cross-sectional areas are given in Table 3. They are also shown graphically in Fig. 1. In Table 2, the natural frequencies in the bracket are analysis results from an optimum design of each different configuration. In Fig. 1, it is interesting to note that whereas the optimum designs for case 2 (simply supported beam) and case 3 (free-free beam) have the general trend of larger cross-sectional areas in the center portion of the beam, this is quite different from the optimum design of case 1 (cantilever beam) that has a tapered cross section. However, when all models are considered in case 4, the optimum design appears to be a compromise of the above cases.

Concluding Remarks

In this paper an approach for minimum weight design of structures with multiple configurations has been developed. The SLP has been implemented. The developed programs are designed to interface with MSC/NASTRAN normal mode analysis runs. The design iteration in the program is carried out in a reduced modal space. The saving of computer times by these efficient reanalysis on normal mode is very significant. Near optimum designs are obtained for the example problem in less than two design cycles.

From the example problem, it is clear that it is essential to consider the various configurations for frequency constraints. With some modification, the method presented here can also be used in structural model parameter identification using data from multiple boundary condition tests.[6]

References

[1]Levy, R., and Choi, K., "Implementation of Natural Frequency Analysis and Optimality Criterion Design," *Computers and Structures,* Vol. 10, No. 1/2, 1979, pp. 277–282.

[2]Pedersen, P., "Design with Several Eigenvalue Constraints by Finite Elements and Linear Programming," *Journal of Structural Mechanics,* Vol. 10, No. 3, 1982–83, pp. 243–271.

[3]Khot, N. S., "Optimization of Structures with Multiple Frequency Constraints," *Computers and Structures,* Vol. 20, No. 5, 1985, pp. 869–876.

[4]Wang, B. P., "Synthesis of Structures with Multiple Frequency Constraints," AIAA Paper 86-0951, May 1986.

[5]Wang, B. P., "Structural Dynamic Optimization Using Reanalysis Techniques," *International Journal of Analytical and Experimental Modal Analysis,* Vol. 2, 1987, pp. 50–58.

[6]Kuo, C. P., and Wada, B. K., "Multiple Boundary Condition Test (MCT): Identification with Mode Shapes," AIAA Paper 88-2353, April 1988.